Methematical statistics

Hypothesis testing

A statistical hypothesis is any guess about the general population distribution, i.e. its functional form or parameter values.

This assumption is tested on the results of a random sample.

The complementary hypotheses are the null hypothesis H_0 and the alternative hypothesis H_1.

Statistical test

A rule of conduct according to which each possible random sample is assigned a decision to reject or not to reject the H0 hypothesis.

Hypothesis testing

Parametric hypotheses - the guess is about the values of the distribution parameters

In other cases - nonparametric hypotheses.

Significance tests

The set of those sample values for which H0 is rejected is called the critical set or the critical area.

The completion of the critical area is called the area of accepting the null hypothesis.

Power of a test

The strength of the test is the probability of not committing a type II error, i.e. rejecting a false null hypothesis, of 1-ß.

The statistical test can be weak or strong:

- strong test - in most cases we are able to reject the false null hypothesis
- weak test - there is a good chance that we will not reject the null hypothesis, despite its falsehood.

Power of a test

Typically, parametric tests are defined as more powerful statistical tests compared to their nonparametric counterparts, therefore, whenever assumptions allow, we choose parametric tests from two tests: parametric vs nonparametric.

Statistical hypothesis testing

- state the relevant null and alternative hypotheses
- consider the statistical assumptions being made about the sample in doing the test (variables independence, normal distribution)
- decide which test is appropriate, and state the relevant test statistic (a Student's t distribution, a normal distribution etc.)
- select a significance level (α), a probability threshold below which the null

Statistical hypothesis testing

	H_0 is true	H_1 is true
Accept Null Hypothesis	Right decision	Wrong decision Type II Error
Reject Null Hypothesis	Wrong decision Type I Error	Right decision

Significance test for the population mean

We construct a variable

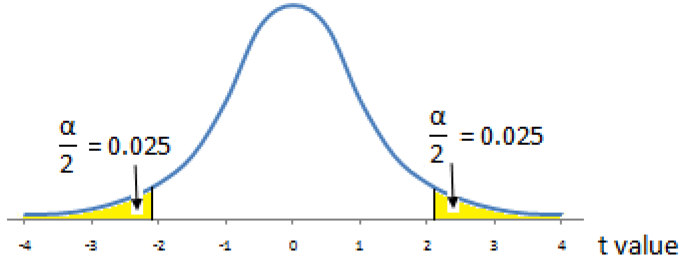
$$t_{emp} = \frac{\bar{x} - \mu_0}{s}\sqrt{n}$$

which has a Student t-distribution with n-1 degrees of freedom.

If H0 is true then t_{emp} should not exceed $t_{\alpha,n-1}$.

Student's t Distribution Table

For example, the t value for 18 degrees of freedom is 2.101 for 95% confidence interval (**2-Tail** α = 0.05).



	90%	95%	97.5%	99%	99.5%	99.95%	**1-Tail Confidence Level**
	80%	90%	95%	98%	99%	99.9%	**2-Tail Confidence Level**
	0.100	0.050	0.025	0.010	0.005	0.0005	**1-Tail Alpha**
df	0.20	0.10	0.05	0.02	0.01	0.001	**2-Tail Alpha**
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	

http://growingknowing.com/GKStatsBookStudentTTable.html

Significance test for the population mean

The content of vitamin C in tomato juice prepared according to the selected recipe was tested. Measurements of vitamin C content in 17 samples gave the results (mg / 100g): 18.8 18.2 16.5 20.5 20.0 20.2 14.6 26.9 25.0 22.3 20.9 22, 1 23.1 13.6 21.4 14.8 17.9. Assume that the tested feature has a normal distribution with unknown parameters and verify the hypothesis that the average vitamin C content in the juice prepared according to this recipe is 20; take a significance level of 0.05.

Significance test for population mean - example

variable: vitamin C content in tomato juice prepared according to a selected recipe

population: tomato juice prepared according to the selected recipe

null hypothesis: the average content of vitamin C in the juice prepared according to this recipe is 20 mg / 100g

or H_0: $\mu=20$

alternative hypothesis: the average content of vitamin C in the juice prepared according to this recipe is 20 mg / 100g

or H_1: $\mu\neq20$

Normality test

Shapiro-Wilk test - a standard test used to test the normality of data

example:

we check if the sample x1, ..., xn comes from the normal distribution

The null and alternative hypothesis in the Shapiro-Wilk test takes the following form:

H0: The sample is from a normally distributed population

H1: The sample is not from a normally distributed population

In order to perform the test, the W statistic is used.

Online calculator

Online calculator for Shapiro-Wilk:

http://www.statskingdom.com/320ShapiroWilk.html

Significance test for population mean - example

We reject the null hypothesis when

$t_{emp} > t_{kryt}$

that is, we went beyond the assumed area of acceptance H0

$p < \alpha$

i.e. the chance of obtaining the obtained temp is lower than the assumed significance level

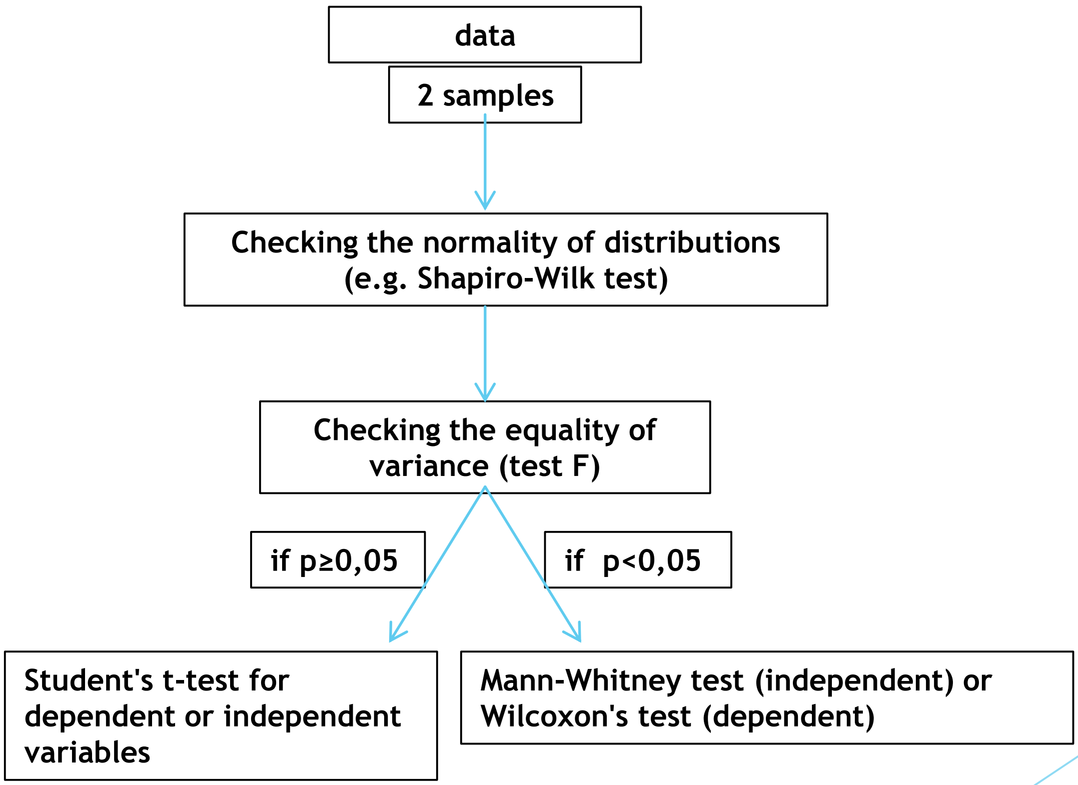
Online calculator

https://www.socscistatistics.com/tests/tsinglesample/default2.aspx

Hypothesis testing

Two-population significance test

Test istotności dla dwóch średnich



Daniel Granato, Verônica Maria de Araújo Calado, Basil Jarvis „Observations on the use of statistical methods in Food Science and Technology"
Food Research International 55 (2014) 137–149

Two mean significance test to infer the equality of two means in two normal populations in the independent variables

We study two general populations with normal distributions $N(\mu_1,\sigma_1)$ and $N(\mu_2, \sigma_2)$.

Additionally, groups are independent, i.e. groups that do not mix with each other.

The condition of the normality of distributions is satisfied - we use the t test for the equality of two means.

Two mean significance test to infer the equality of two means in two normal populations in the independent variables

It is checked if $\mu_1 = \mu_2$:

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

Significance test for two means

$$t_{emp} = \frac{\overline{x_1} - \overline{x_2}}{s_r}$$

$$s_r = \sqrt{s_e^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$s_e^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$\left|t_{emp}\right| > t_{\alpha,n_1+n_2-2}$$

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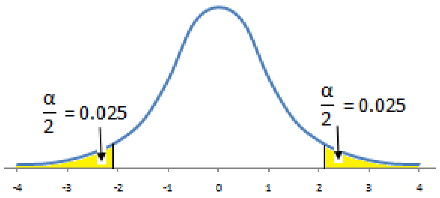

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Two means equality test for two dependent groups

Most often, these are exactly the same units, e.g. patients whose reaction time to a stimulus is observed before and after drug administration.

The analyzed groups are therefore dependent / related.

The question: does the administration of the drug change the patients' reaction time or does the reaction time before drug administration on average equal the reaction time after drug administration?

Two means equality test for two dependent groups

The test examines the significance of the difference of means for the variables.

It is checked if $\mu_1 = \mu_2$:

H_0: $\mu_1 - \mu_2 = 0$

H_1: $\mu_1 - \mu_2 \neq 0$

Test t - example

Fruit weight tests were carried out on two cultivars of field cucumbers: Przybyszewski (A) and Warszawski Gruntowy (B). The data comes from normal distributions. The following measurements were obtained:

A 150 163 155 164 162 170 174 168 172 171 168

B 152 159 161 164 169 177 173 171 176 176 174

Does variety B significantly differ from variety A in terms of average fruit weight? Verify the appropriate hypothesis at the significance level $\alpha = 0.05$.

Test t – przykład (próby niezależne, spełniony warunek o normalności rozkładów)

H_0: $\mu_1 = \mu_2$

The cucumber varieties tested did not differ in the average weight.

H_1: $\mu_1 \neq \mu_2$

The cucumber varieties tested did not differ in the average weight.

Test t - example

the value of the test statistic

t = -0.95

p value

p = 0.35

p> α

there is no reason to reject the null hypothesis

The cucumber varieties tested did not differ in the average weight.

Test t – example online calculator

https://www.socscistatistics.com/tests/studentttest/default.aspx

Please choose the two-sided test, i.e. we are interested in whether the averages differ, and not whether Przybyszewski has an average weight greater than the Warsaw Land.

Test t for dependent samples

https://www.socscistatistics.com/tests/ttestdependent/default2.aspx

Nonparametric tests

If the condition of the normality of distributions is not met, especially for samples, the number of which does not exceed 30, it is not justified to use the t-test for equality of two means.
If the assumption of normality is not met, the equality of means can be verified with a non-parametric test.

Nonparametric tests

The Mann-Whitney test is an example of a nonparametric test quite commonly used in the context of nonparametric comparison of means for two independent groups.

Nonparametric tests

Wilcoxon signed pair test - equivalent to two paired samples t-test.

U Manna-Whitney test

It is used to test the hypothesis that two independent samples are from identical populations.
It replaces the Student's t-test for two means when the required assumptions are not met.

U Manna-Whitneya test example

Infestation by viral diseases of two potato cultivars (A and B) was assessed. The evaluation was carried out on a 5-point scale (1- no diseases .... 5- very strong paralysis by viral diseases). The following results were obtained on the basis of a dozen or so plants selected at random for each variety:

Odmiana	A	A	A	A	A	A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	B	B	B	B	B	B	B
Stopień porażenia	3	1	1	2	3	3	4	4	5	1	1	1	1	2	2	1	2	3	3	4	4	5	3	2	1	4	1	5

U Manna-Whitney test calculator

https://www.socscistatistics.com/tests/mannwhitney/

Wilcoxon test calculator

https://www.socscistatistics.com/tests/signedranks/default.aspx