INTEGRALS

The prime function $f(x)$ in the interval $a < x < b$ is any such function $F(x)$, whose derivative $F'(x)$ is equal to this function.

Two functions that have the same derivative in the same interval can differ by a constant.

Indefinite integral of the function $f(x)$ after dx:

$$\int f(x)dx$$

If

$\int f(x)dx = F(x) + C$,

then the function $F(x)$ is called the original function, and C is called the constant and

$F'(x) = f(x)$.

Constant C:

If the derivative of a function is $3x^2$, then the function can be $x^3 + 4$ or $x^3 - 1$ or generally $x^3 + C$.

Calculus formulas:

- $\int x^a dx = \frac{1}{a+1} x^{a+1} + C, a \neq -1$

e.g.

$$\int dx = x + C$$

$$\int \frac{1}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} dx = \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + C = 2x^{\frac{1}{2}} + C$$

$$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{1}{-2+1} x^{-2+1} + C = -x^{-1} + C = -\frac{1}{x} + C$$

but

- $\int \frac{1}{x} dx = ln|x| + C$

- $\int e^x dx = e^x + C$

- $\int a^x dx = \frac{a^x}{lna} + C$

Properties of integrals:

$$\int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int k f(x) dx = k \int f(x) dx$$

Examples:

$$\int \frac{\sqrt{x}-\sqrt[3]{x}}{x^2}dx = \int \left(x^{\frac{1}{2}-2} - x^{\frac{1}{3}-2}\right)dx = \int \left(x^{-\frac{3}{2}} - x^{-\frac{5}{3}}\right)dx =$$

$$= \int x^{-\frac{3}{2}}dx - \int x^{-\frac{5}{3}}\,dx = \frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1} - \frac{1}{-\frac{5}{3}+1}x^{-\frac{5}{3}+1} + C =$$

$$= -2x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{2}{3}} + C$$

DEFINITE INTEGRAL

If in the interval $< a, b >$ there is $f(x) > 0$, then the area bounded by the curve of the curve $y = f(x)$, a segment of the axis Ox and the lines $x = a$ and $x = b$ is equal to the definite integral:

$$\int_a^b f(x)dx$$

If in the interval $< a, b >$ there is $f(x) \leq 0$, then the area bounded by the curve of the curve $y = f(x)$, a by the segment of the Ox axis and the lines $x = a$ and $x = b$ is equal to the definite integral:

$$-\int_a^b f(x)dx$$

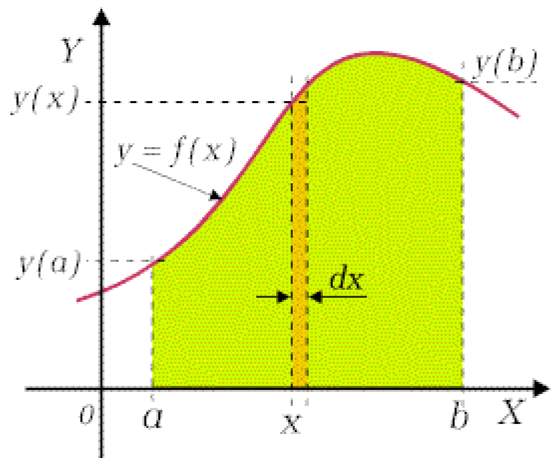
If $F(x)$ is a prime function $f(x)$, continuous in the interval $< a, b >$, i.e.

$F'(x) = f(x)$, then:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(x)|_a^b = F(b) - F(a)$$

the difference $F(b) - F(a)$ does not depend on the integration constant C.

Graphic interpretation



http://www.if.pw.edu.pl/~wosinska/am2/matma/calka/calka.HTM

Properties of definite integrals:

- if $a \leq b \leq c$ there is additivity of the integrals to the integration interval

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

- the constant factor can be switched off before the sign of the definite integral

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

the integral of the sum equals the sum of the integrals

$$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Examples:

$$\int_{-1}^{2} (x^3 - 2x + 1)dx = \int_{-1}^{2} x^3 dx - 2\int_{-1}^{2} xdx + \int_{-1}^{2} dx =$$

$$= \frac{1}{4}x^4\Big|_{-1}^{2} - 2 \cdot \frac{1}{2}x^2\Big|_{-1}^{2} + x|_{-1}^{2} =$$

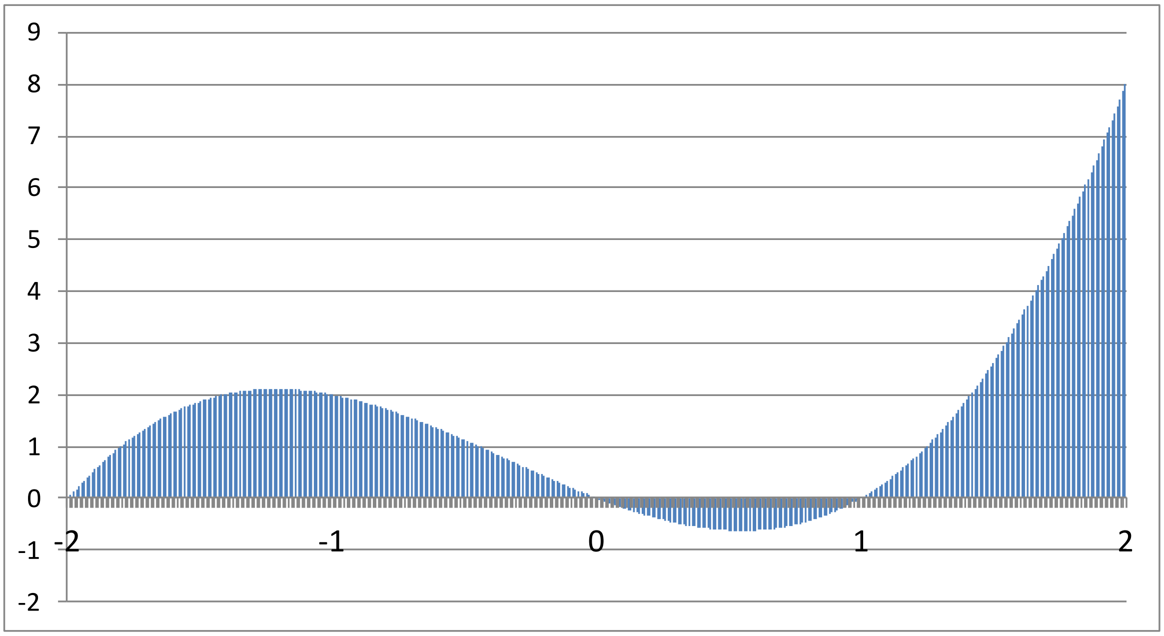
$$= \frac{1}{4}(2^4 - (-1)^4) - (2^2 - (-1)^2) + \big(2 - (-1)\big) =$$

$$= \frac{15}{4} - 3 + 3 = \frac{15}{4}$$

$$\int_0^1 \frac{1}{\sqrt{x}}dx = \int_0^1 x^{-\frac{1}{2}}dx = 2x^{\frac{1}{2}}\Big|_0^1 = 2\sqrt{x}\Big|_0^1 = 2 - 0 = 2$$

Calculating the area under the curve

Calculate the area of the area bounded by the curve of the curve $y = x^3 + x^2 - 2x$, axle distance Ox and functions $x = -2$ and $x = 2$.



$$\int_{-2}^{2}(x^3+x^2-2x)dx=$$

$$=\int_{-2}^{0}(x^3+x^2-2x)dx+\int_{0}^{1}(x^3+x^2-2x)dx+\int_{1}^{2}(x^3+x^2-2x)dx$$

$$\neq$$

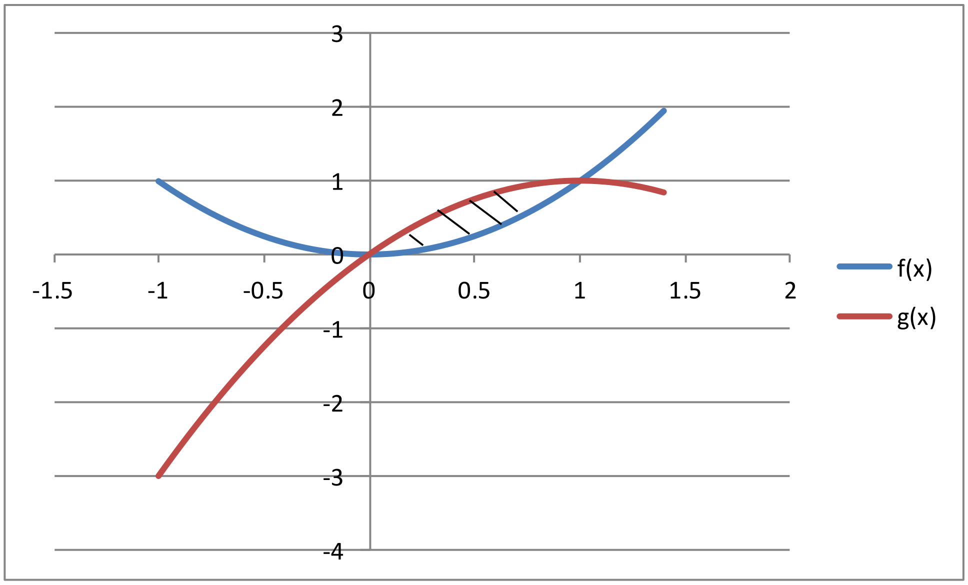
$$\int_{-2}^{0}(x^3+x^2-2x)dx-\int_{0}^{1}(x^3+x^2-2x)dx+\int_{1}^{2}(x^3+x^2-2x)dx=$$

$$=\left[\frac{1}{4}x^4+\frac{1}{3}x^3-x^2\right]_{-2}^{0}-\left[\frac{1}{4}x^4+\frac{1}{3}x^3-x^2\right]_{0}^{1}+\left[\frac{1}{4}x^4+\frac{1}{3}x^3-x^2\right]_{1}^{2}=$$

$$=\left(0-\left(\frac{16}{4}-\frac{8}{3}-4\right)\right)-\left(\left(\frac{1}{4}+\frac{1}{3}-1\right)-0\right)+\left(\left(\frac{16}{4}+\frac{8}{3}-4\right)-\left(\frac{1}{4}+\frac{1}{3}-1\right)\right)$$

$$=\frac{8}{3}+\frac{5}{12}+\frac{37}{12}=\frac{37}{6}$$

Determine the area of the area bounded by the parabolas graphs $f(x)=x^2$ and $g(x)=2x-x^2$.



We determine the points of intersection of parabolas by solving the equation

$2x - x^2 = x^2$

$2x - 2x^2 = 0$

$x = 0$ lub $x = 1$

Thus:

$$\int_0^1 (2x - 2x^2)dx = 2\int_0^1 (x - x^2)dx = 2(\frac{1}{2}x^2 - \frac{1}{3}x^3)\Big|_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$